Math 60: Appendix C.2 Word Problems

1) A manufacturer makes three different models of swing sets. The Monkey takes 2 hours to cut the wood, 2 hours to stain, and 3 hours to assemble. The Gorilla takes 3 hours to cut the wood, 4 hours to stain, and 4 hours to assemble. The King Kong takes 4 hours to cut the wood, 5 hours to stain, and 5 hours to assemble. The company has 61 hours available to cut the wood, 73 hours available to stain, and 83 hours available to assemble. How many of each type of swing set can be manufactured?

x = # Monkey
y = # Gorilla
z = # King Kong

Cut wood $\underset{\text{per Monkey}}{2 \text{ hrs}} \cdot \underset{\text{Monkeys}}{x} = \underset{\text{\# hrs cutting wood for Monkeys.}}{2x}$

Eqn for cutting wood:

$\underbrace{2x}_{\text{Monkey hrs}} + \underbrace{3y}_{\text{Gorilla hrs}} + \underbrace{4z}_{\text{KK hrs}} = 61$ ← total hrs for cutting

Ditto stain: $2x + 4y + 5z = 73$

Ditto assemble: $3x + 4y + 5z = 83$

Solve 3x3 system

$$\begin{cases} 2x + 3y + 4z = 61 \\ 2x + 4y + 5z = 73 \\ 3x + 4y + 5z = 83 \end{cases}$$

(see additional sheets)

Solution $(10, 7, 5)$

$\boxed{\begin{array}{l} \text{10 Monkey} \\ \text{7 Gorilla} \\ \text{5 King Kong} \end{array}}$

2) The function $f(x) = ax^2 + bx + c$ is a quadratic function, where a, b, and c are constants.

a. If $f(-1) = 6$, then $6 = a(-1)^2 + b(-1) + c$ (by substituting x and y), which gives the equation $a - b + c = 6$. Write two additional equations using a, b, and c, if $f(1) = 2$ and $f(2) = 9$

b. Use the three equations in three unknown variables from part a, solve this system to determine the coefficients a, b, and c, then write the function.

a) $f(-1) = 6$
$a(-1)^2 + b(-1) + c = 6$
$\underline{\underline{a - b + c = 6}}$

$f(1) = 2$
$a(1)^2 + b(1) + c = 2$
$\underline{\underline{a + b + c = 2}}$

$f(2) = 9$
$a(2)^2 + b(2) + c = 9$
$\underline{\underline{4a + 2b + c = 9}}$

$$\boxed{\begin{cases} a - b + c = 6 \\ a + b + c = 2 \\ 4a + 2b + c = 9 \end{cases}}$$

b) Solve system
(see additional sheets)

solution
$a = 3$
$b = -2$
$c = 1$

function
$\boxed{f(x) = 3x^2 - 2x + 1}$

3) An application of Kirchhoff's Rule to a circuit results in the following system of equations, where i_1, i_2, and i_3 are the currents in three circuits. Solve the system to find the currents i_1, i_2, and i_3.

$$\begin{cases} -3-3i_1+2i_3=0 \\ -22+4i_2+2i_3=0 \\ i_1+i_3=i_2 \end{cases}$$

You do not need to understand the circuit or Kirchhoff's rule -- just solve the system.

$\left\{\begin{array}{l} i_1 \text{ is like } x \\ i_2 \text{ is like } y \\ i_3 \text{ is like } z \end{array}\right\}$ If you want to temporarily change to x, y, z, do. But give answers using i_1, i_2, i_3.

This system is disorganized.

$$\begin{array}{l} -3-3x+2z=0 \\ +3 \qquad\qquad\quad +3 \\ \hline -3x+2z=3 \\ \underline{\underline{-3x \qquad +2z=3}} \end{array}$$

$$\begin{array}{l} x+z=y \\ \quad -y \quad -y \\ \hline \underline{\underline{x-y+z=0}} \end{array}$$

$$\begin{array}{l} -22+4y+2z=0 \\ +22 \qquad\qquad +22 \\ \hline 4y+2z=22 \end{array}$$

* can divide all by 2! *

$$\underline{\underline{2y+z=11}}$$

Solve system
(see additional sheets)

$$\begin{cases} -3x \qquad +2z=3 \\ \quad 2y+z=11 \\ x-y+z=0 \end{cases} \Rightarrow (1,4,3)$$

$$\boxed{i_1=1 \quad i_2=4 \quad i_3=3}$$

4) Antonio is on a special diet that requires he consume 1325 calories, 172 grams of carbohydrates, and 63 grams of protein for lunch. He wishes to have servings of Broccoli and Cheese Baked Potato ("'tater"), Chicken BLT Salad ("salad"), and a medium Coke. Each 'tater has 480 calories, 80 g of carbs, and 9 g of protein. Each salad has 310 calories, 10 g of carbs, and 33 g of protein. Each Coke has 140 calories, 37 g of carbs, and 0 g of protein. How many servings of each does Antonio need?

x = servings of tater
y = servings of salad
z = servings of Coke

Calories from 'taters:

$$\underset{\text{calories per 'tater serving}}{480} \times \underset{\text{\# tater servings}}{x} = \underset{\text{calories from 'taters}}{480x}$$

Eq'n for calories

$$\underset{\text{calories 'tater}}{480x} + \underset{\text{calories salad}}{310y} + \underset{\text{calories Coke}}{140z} = \underset{\text{total calories}}{1325}$$

$\div 5$: $96x+62y+28z=265$

Eq'n for carbs

$$\underset{\text{carbs tater}}{80x} + \underset{\text{carbs salad}}{10y} + \underset{\text{carbs Coke}}{37z} = \underset{\text{total carbs}}{172}$$

Eq'n for protein

$$\underset{\text{protein tater}}{9x} + \underset{\text{protein salad}}{33y} + \underset{\text{protein Coke}}{0z} = \underset{\text{total protein}}{63}$$

Solve system

$$\begin{cases} 96x+62y+28z=265 \\ 80x+10y+37z=172 \\ 9x+33y \qquad\quad =63 \end{cases}$$

(see additional sheets)

$\boxed{\begin{array}{l}\text{1.5 servings of tater} \\ \text{1.5 servings of salad} \\ \text{1 serving of Coke}\end{array}}$

Math 60 App C.2 (Day 3)

1. See handout for set-up of word problem.
Solve system by elimination
$$\begin{cases} 2x + 3y + 4z = 61 & (A) \\ 2x + 4y + 5z = 73 & (B) \\ 3x + 4y + 5z = 83 & (C) \end{cases}$$

I choose to eliminate x: $(A) + (B) \times (-1)$

$(A) \times 3 + (C) \times (-2)$

$$\begin{array}{rl} 2x + 3y + 4z = 61 & (A) \\ -2x - 4y - 5z = -73 & (B) \times (-1) \\ \hline -y - z = -12 & \text{mult all by } -1 \\ y + z = 12 & (D) \end{array}$$

$$\begin{array}{rl} 6x + 9y + 12z = 183 & (A) \times 3 \\ -6x - 8y - 10z = -166 & (C) \times (-2) \\ \hline y + 2z = 17 & (E) \end{array}$$

Solve 2x2 system (D) and (E)

$$\begin{cases} y + z = 12 & (D) \\ y + 2z = 17 & (E) \end{cases}$$

$$\begin{array}{rl} -y - z = -12 & (D) \times (-1) \\ y + 2z = 17 & \\ \hline z = 5 & \end{array}$$

subst → (D)

$y + 5 = 12$

$y = 7$

subst → (A)

$2x + 3(7) + 4(5) = 61$

$2x + 21 + 20 = 61$

$2x + 41 = 61$

$2x = 20$

$x = 10$

$x = 10$ Monkey sets

$y = 7$ Gorilla sets

$z = 5$ King Kong sets

Math 60 App C.2 (Day 3)

(2) See handout for set-up of word problem.
Solve system by elimination
$$\begin{cases} a-b+c=6 & (D) \\ a+b+c=2 & (E) \\ 4a+2b+c=9 & (F) \end{cases}$$

I choose to eliminate **b** using (D)+(E)
(D)×(2)+(F)

$$\begin{array}{rl} a-b+c=6 & (D) \\ a+b+c=2 & (E) \\ \hline 2a+2c=8 & \end{array}$$

div all by 2

$$\underline{\underline{a+c=4}} \quad (G)$$

$$\begin{array}{rl} 2a-2b+2c=12 & (D)\times 2 \\ 4a+2b+c=9 & (F) \\ \hline 6a+3c=21 & \end{array}$$

div all by 3

$$\underline{\underline{2a+c=7}} \quad (H)$$

Solve 2×2 system:
$$\begin{cases} a+c=4 & (G) \\ 2a+c=7 & (H) \end{cases}$$

$$\begin{array}{rl} -a-c=-4 & (G)\times(-1) \\ 2a+c=7 & \\ \hline a=3 & \end{array}$$

subst ⇒ (G)
$3+c=4$
$c=1$

subst ⇒ (D)
$3-b+1=6$
$-b+4=6$
$-b=2$
$b=-2$

$$\boxed{\begin{array}{l} a=3 \\ b=-2 \\ c=1 \end{array}}$$

$f(x)=ax^2+bx+c$

so

$$\boxed{f(x)=3x^2-2x+1}$$

Math 60 App C.3 Day 3

3 See handout for set-up of word problem.

Solve system by elimination:
$$\begin{cases} -3x + 2z = 3 & \text{A} \\ 2y + z = 11 & \text{B} \\ x - y + z = 0 & \text{C} \end{cases}$$

I choose eliminate x: A + C × 3 and B

$$\begin{array}{rrrl} -3x & & +2z = 3 & \text{A} \\ 3x & -3y & +3z = 0 & \text{C} \times 3 \\ \hline & -3y & +5z = 3 & \text{D} \end{array}$$

Solve 2×2 system

$$\begin{cases} -3y + 5z = 3 & \text{D} \\ 2y + z = 11 & \text{B} \end{cases}$$

eliminate z:

$$\begin{array}{rrl} -3y + 5z = 3 & & \text{D} \\ -10y - 5z = -55 & & \text{B} \times (-5) \\ \hline -13y = -52 \end{array}$$

$$\frac{-13y}{-13} = \frac{-52}{-13}$$

$$y = 4$$

subst ⇒ B

$2(4) + z = 11$

$8 + z = 11$

$z = 3$

Subst ⇒ C

$x - 4 + 3 = 0$

$x - 1 = 0$

$x = 1$

Remember, we used variables x, y and z because they're easier, but

$$\boxed{\begin{array}{l} x \to i_1 = 1 \\ y \to i_2 = 4 \\ z \to i_3 = 3 \end{array}}$$

Math 60 App C.3 Day 3

(4) See handout for set-up of word problem.

Solve system by elimination
$$\begin{cases} 96x + 62y + 28z = 265 & (A) \\ 80x + 10y + 37z = 172 & (B) \\ 9x + 33y = 63 & (C) \end{cases}$$

I choose

eliminate z: (A) $\times 37$ + (B) $\times (-28)$

(C)

$$\begin{array}{rl} 3552x + 2294y + 1036z = 9805 & (A) \times 37 \\ -2240x - 280y - 1036z = -4816 & (B) \times (-28) \\ \hline 1312x + 2014y = 4989 & (D) \end{array}$$

Solve 2x2 system

$$\begin{cases} 1312x + 2014y = 4989 & (D) \\ 9x + 33y = 63 & (C) \end{cases}$$

eliminate x:

$$\begin{array}{rl} 11808x + 18126y = 44901 & (D) \times 9 \\ -11808x - 43296y = -82656 & (C) \times (-1312) \\ \hline -25170y = -37755 & \end{array}$$

$$\frac{-25170y}{-25170} = \frac{-37755}{-25170}$$

$$y = 1.5$$

subst $\Rightarrow$ (C)

$$9x + 33(1.5) = 63$$
$$9x + 49.5 = 63$$
$$\frac{9x}{9} = \frac{13.5}{9}$$
$$x = 1.5$$

subst $\Rightarrow$ (B)

$$80(1.5) + 10(1.5) + 37z = 172$$
$$120 + 15 + 37z = 172$$
$$37z = 37$$
$$z = 1$$

$x = 1.5$ servings tater

$y = 1.5$ servings salad

$z = 1$ serving Coke